Radiative effects on non-LTE hydrogen level populations

Andri Prozesky

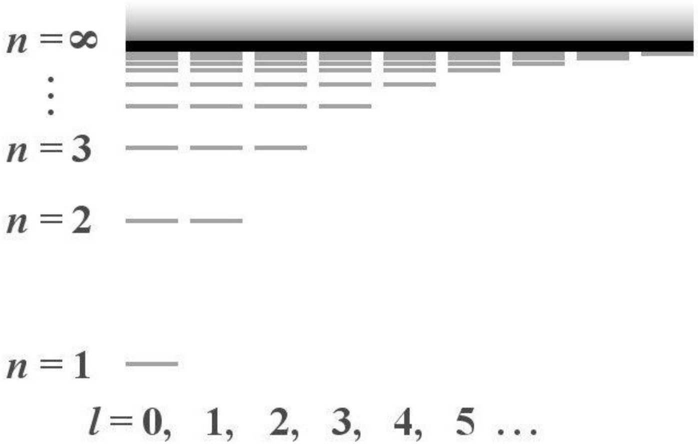
Outline

- Updated departure coefficients for H
  - Stopping criterion for iterative methods
  - Continuum radiation fields

Prozesky & Smits, 2018, MNRAS, 478, 2766

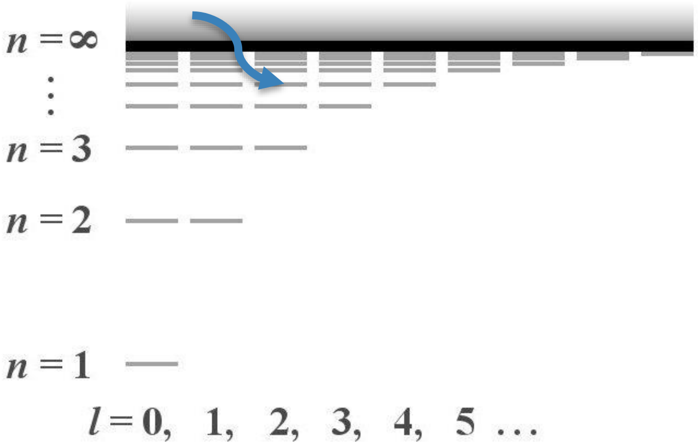
Capture-collision-cascade

- Photoionized gas (HII regions, PNe)
- Interaction between ions and free electrons



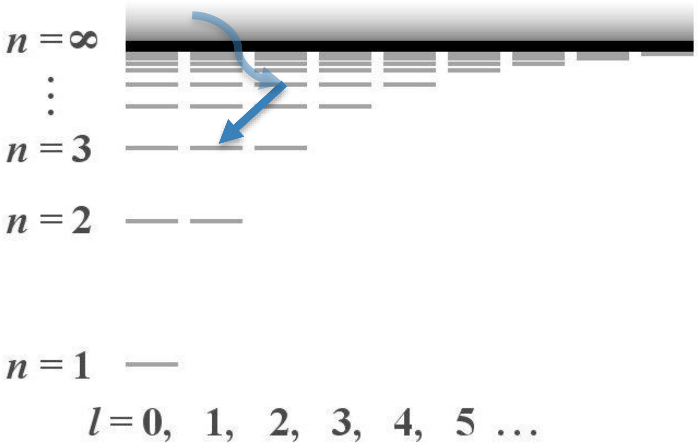
Capture-collision-cascade

- Photoionized gas (HII regions, PNe)
- Interaction between ions and free electrons



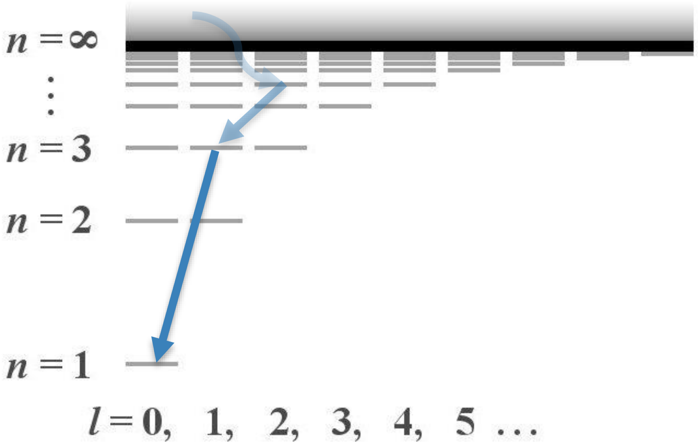
Capture-collision-cascade

- Photoionized gas (HII regions, PNe)
- Interaction between ions and free electrons



Capture-collision-cascade

- Photoionized gas (HII regions, PNe)
- Interaction between ions and free electrons



Departure coefficients

- Saha-Boltzmann equation (TE)

$$N_{nl}^{\mathrm{TE}} = N_e N_i \frac{g_{nl}}{g_e g_i} \left(\frac{h^2}{2\pi m_e k_B T} \right)^{3/2} e^{\chi_{nl}/kT}$$

Departure coefficients

- Saha-Boltzmann equation (TE)

$$N_{nl}^{\mathrm{TE}} = N_e N_i \frac{g_{nl}}{g_e g_i} \left(\frac{h^2}{2\pi m_e k_B T} \right)^{3/2} e^{\chi_{nl}/kT}$$

- Deviation from TE

$$N_{nl} = b_{nl} \times N_{nl}^{\mathrm{TE}}$$

Departure coefficients

- Saha-Boltzmann equation (TE)

$$N_{nl}^{\mathrm{TE}} = N_e N_i \frac{g_{nl}}{g_e g_i} \left(\frac{h^2}{2\pi m_e k_B T} \right)^{3/2} e^{\chi_{nl}/kT}$$

- Deviation from TE

$$N_{nl} = b_{nl} \times N_{nl}^{\mathrm{TE}}$$

Departure coefficients

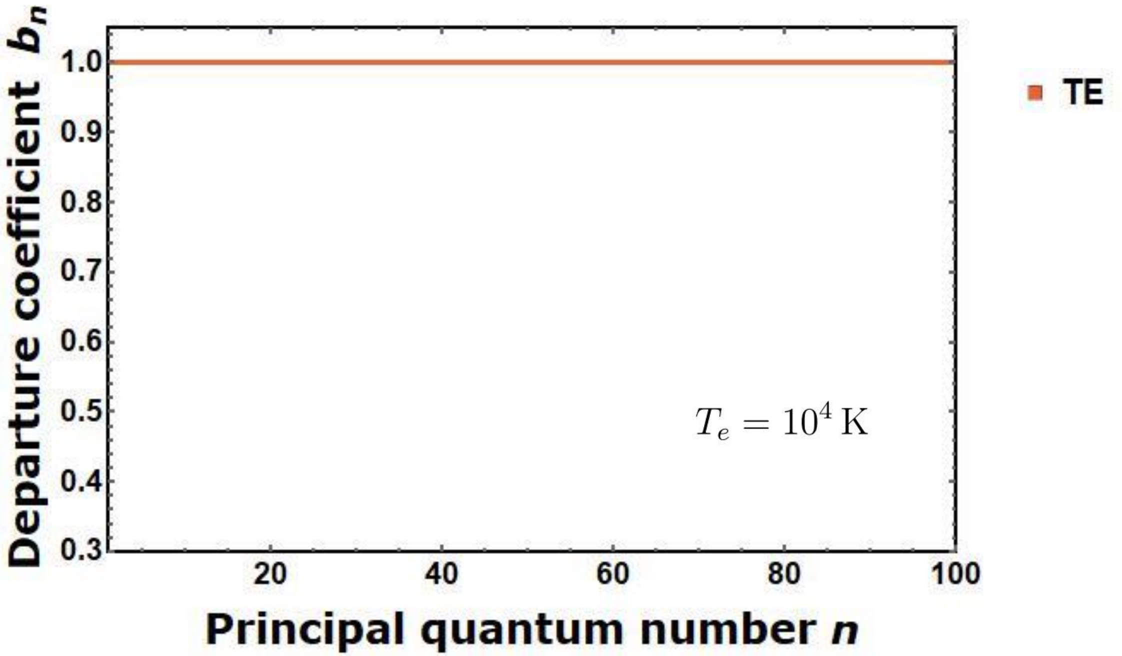
- Saha-Boltzmann equation (TE)

$$N_{nl}^{\mathrm{TE}} = N_e N_i \frac{g_{nl}}{g_e g_i} \left(\frac{h^2}{2\pi m_e k_B T} \right)^{3/2} e^{\chi_{nl}/kT}$$

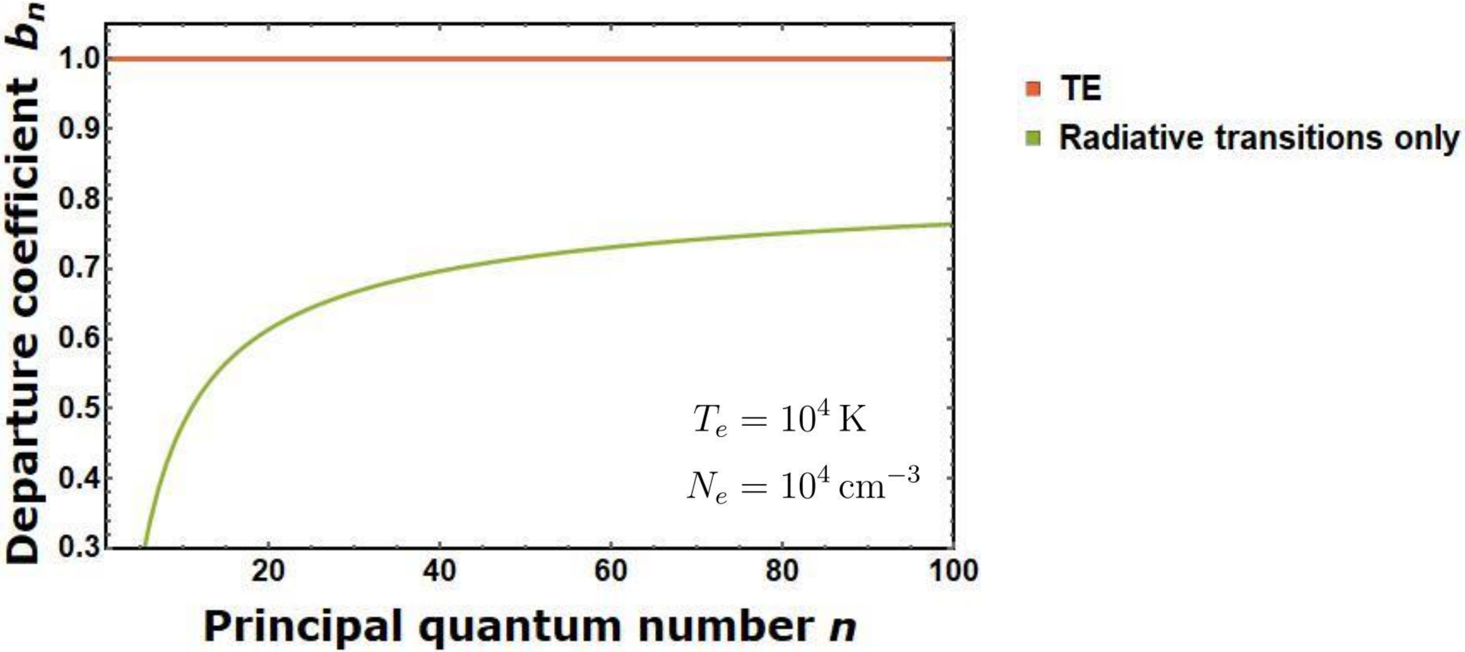
- Deviation from TE

$$N_{nl} = b_{nl} \times N_{nl}^{\mathrm{TE}}$$

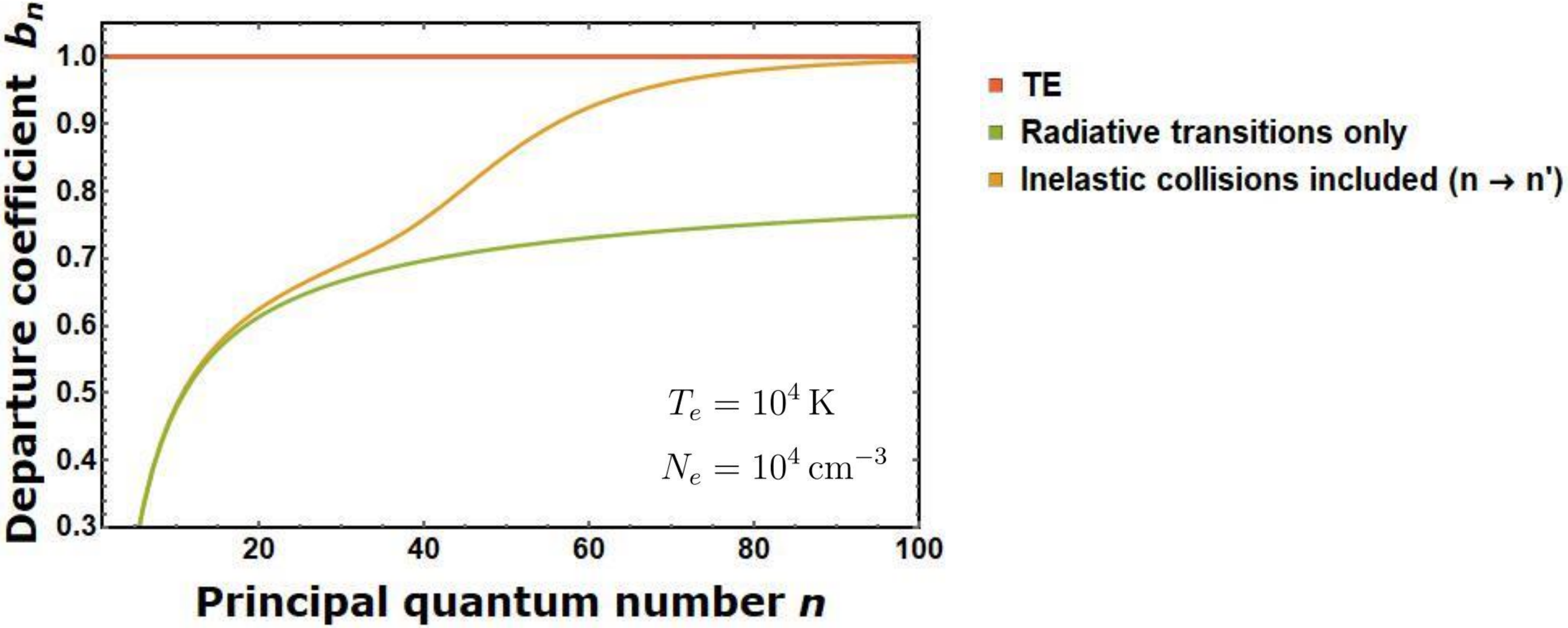
Departure coefficients



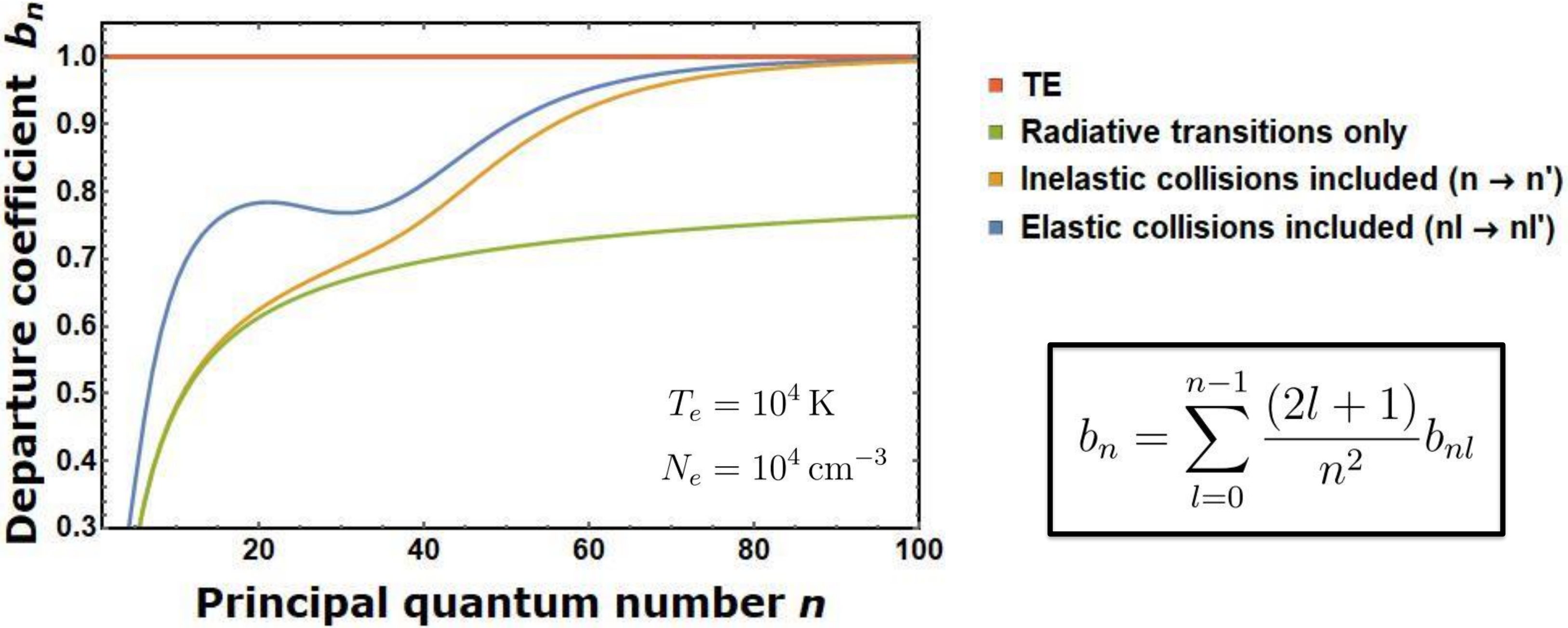
Departure coefficients



Departure coefficients



Departure coefficients



$$b_n = \sum_{l=0}^{n-1} \frac{(2l+1)}{n^2} b_{nl}$$

Calculating b_{nl}'s

- Statistical balance equations

  (all rates into nl) = (all rates out of nl)

- Linear set of $\mathcal{O}\left(10^4\right)$ equations

- Iterative solvers
  - Stopping criterion

Stopping criterion

$$S_n = \sum_{k=2}^{n} \frac{1}{k\,(\ln k)^2}$$

$$S_\infty = 2.1097\ldots$$

Stopping criterion

$$S_n = \sum_{k=2}^{n} \frac{1}{k\,(\ln k)^2}$$

$$S_\infty = 2.1097\ldots$$

Requirement:

$$\frac{S_\infty - S_n}{S_\infty} < \epsilon = 10^{-6}$$

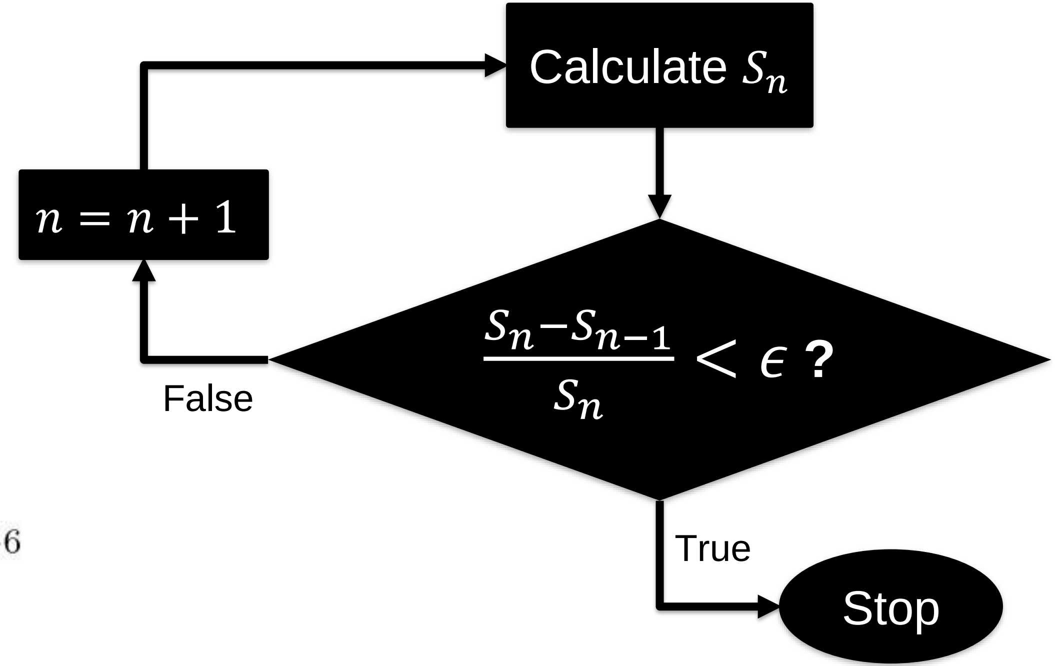
Stopping criterion

$$S_n = \sum_{k=2}^{n} \frac{1}{k\,(\ln k)^2}$$

$$S_\infty = 2.1097\ldots$$

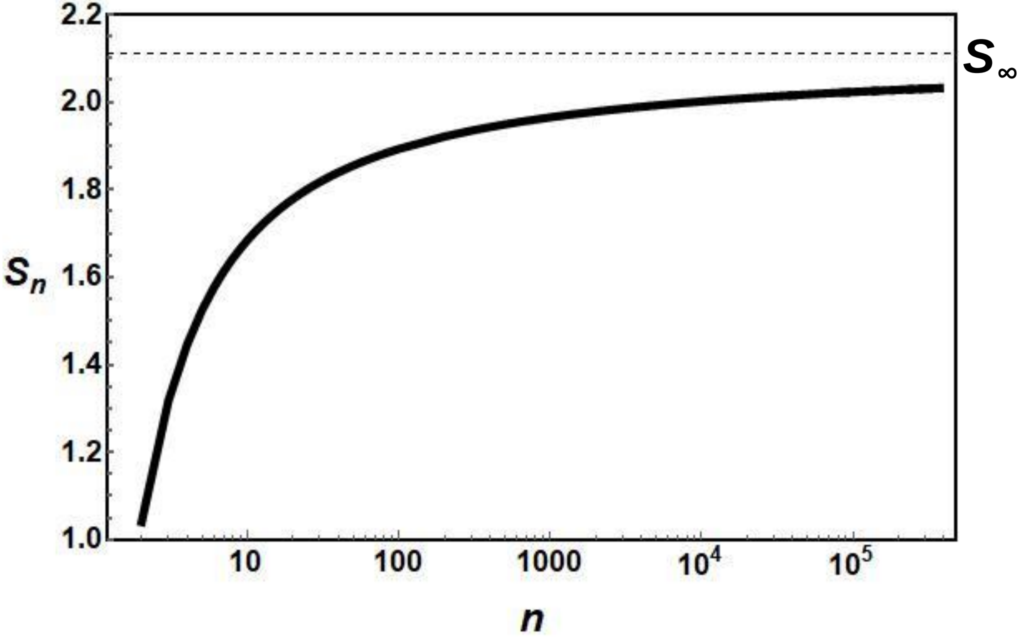
Requirement:

$$\frac{S_\infty - S_n}{S_\infty} < \epsilon = 10^{-6}$$



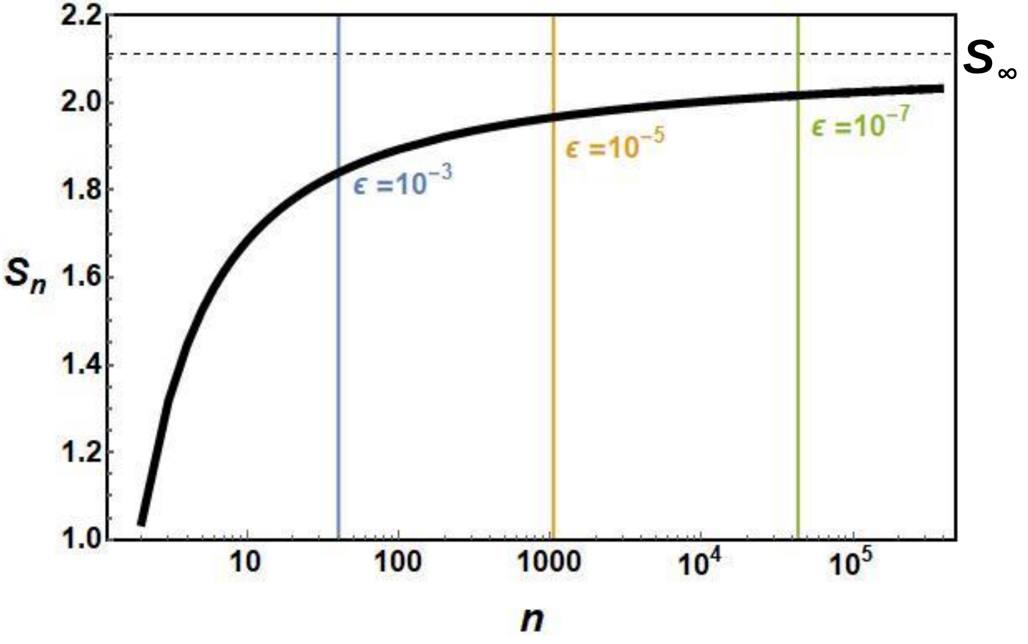
Stopping criterion

$$\frac{S_n - S_{n-1}}{S_n} < \epsilon$$



Stopping criterion

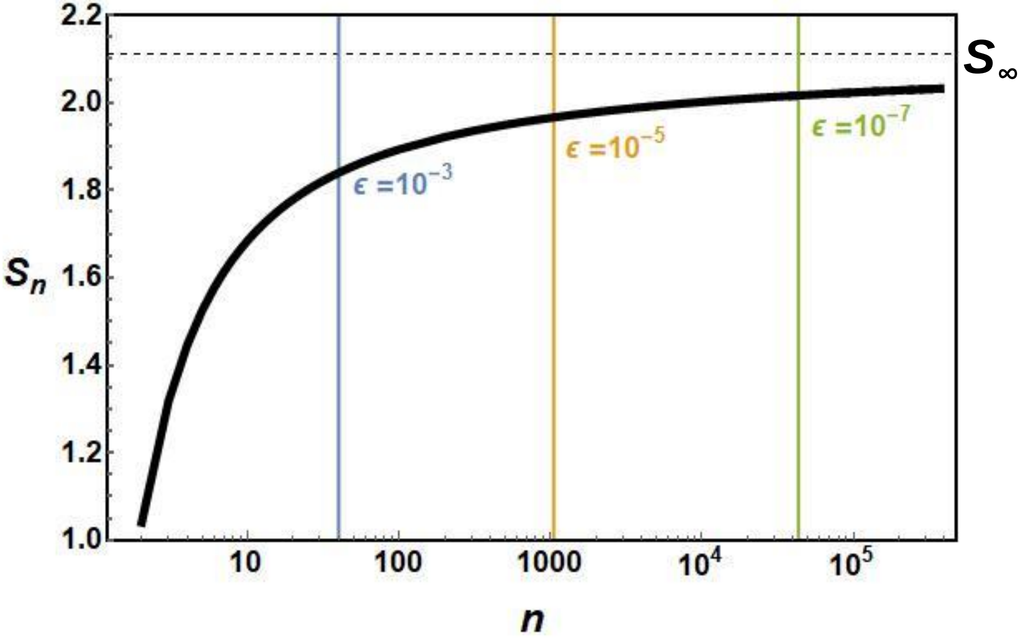
$$\frac{S_n - S_{n-1}}{S_n} < \epsilon$$



Stopping criterion

$$\frac{S_n - S_{n-1}}{S_n} < \epsilon$$

ϵ	Actual error
10^{-3}	0.1272
10^{-5}	0.06811
10^{-7}	0.04438



Back to b_{nl}'s

- Cast equations for b_{nl}'s into matrix form:

$$\mathbf{A} \cdot \mathbf{b} = \mathbf{y}$$

Back to b_{nl}'s

- Cast equations for b_{nl}'s into matrix form:

$$\mathbf{A} \cdot \mathrm{b} = \mathbf{y}$$

Error: $\mathrm{e}^{(i)} = \mathrm{b}^{(i)} - \mathrm{b}$

Back to b_{nl}'s

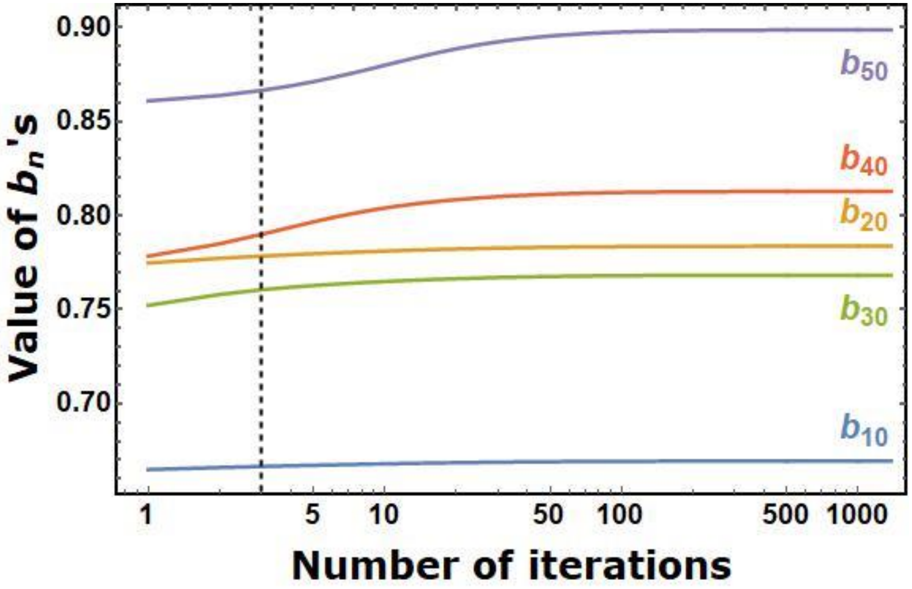
- Cast equations for b_{nl}'s into matrix form:

$$\mathbf{A} \cdot \mathbf{b} = \mathbf{y}$$

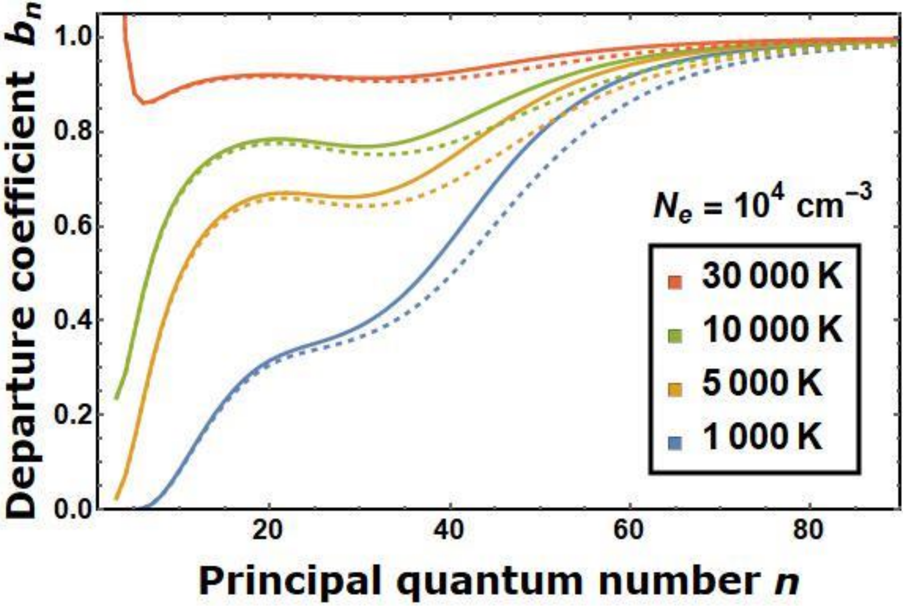
Error: $\mathbf{e}^{(i)} = \mathbf{b}^{(i)} - \mathbf{b}$

$$f(\mathbf{A}^{-1}, \mathbf{y}, \mathbf{b}^{(i)}) \leq \epsilon$$

Stopping criterion



Stopping criterion



- - - Storey & Hummer, 1995, MNRAS, 272, 41

$$b_n = \sum_{l=0}^{n-1} \frac{(2l+1)}{n^2} b_{nl}$$

Continuum radiation fields

$$N_n A_{nn'} \rightarrow N_n A_{nn'} + J_\nu \left(N_n B_{nn'} - N_{n'} B_{n'n} \right)$$

Spontaneous emission

Stimulated emission

Absorption

$$N_e N_i \alpha_n^r \rightarrow N_e N_i \alpha_n^r + J_\nu \left(N_e N_i \alpha_n^s - N_n \alpha_n^p \right)$$

Radiative recombination

Stimulated recombination

Photoionization

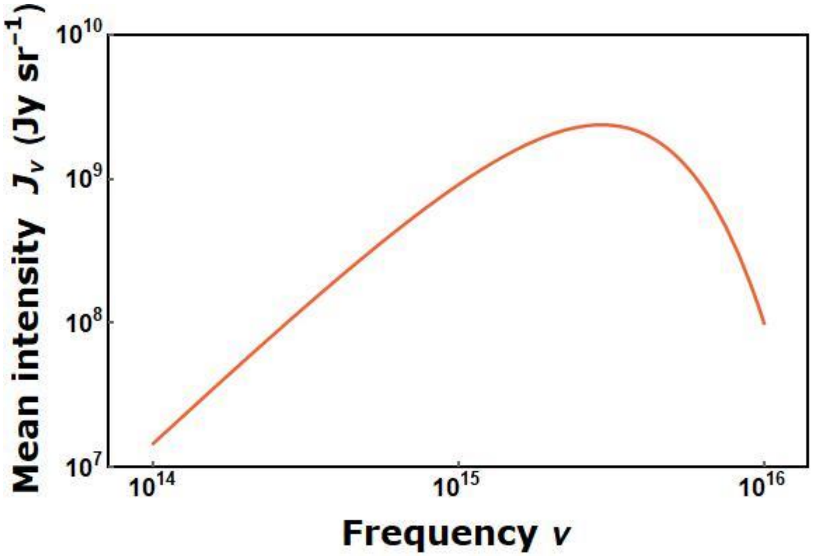
Continuum radiation fields

- ☐ Stellar radiation
- ☐ CMBR
- ☐ Dust
- ☐ Free-free radiation

} J_ν

- Assume gas is optically thin to diffuse radiation

Stellar radiation

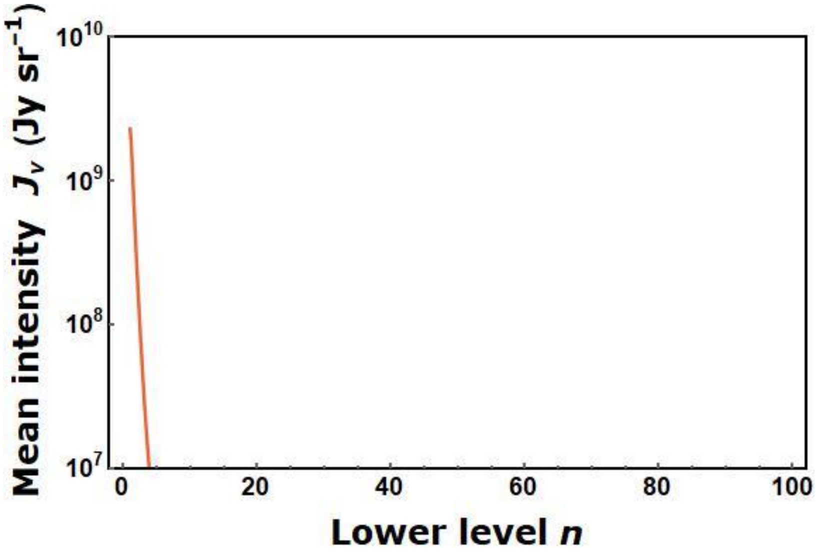


$$T_\star = 50\,000\,\mathrm{K}$$

$$W = 10^{-12}$$

$$d \approx 0.1\,\mathrm{pc}$$

Stellar radiation



$$T_\star = 50\,000\,\mathrm{K}$$

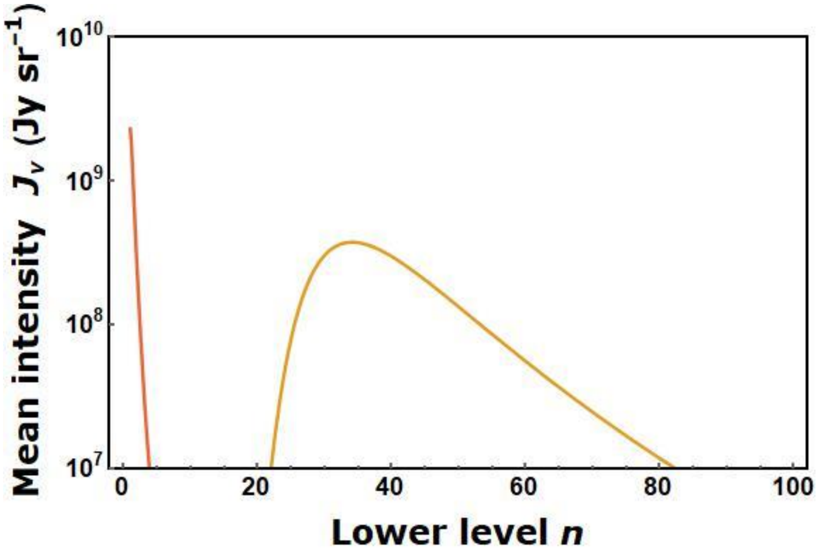
$$W = 10^{-12}$$

$$d \approx 0.1\,\mathrm{pc}$$

$$\mathrm{H}n\alpha$$

$$n + 1 \rightarrow n$$

CMB



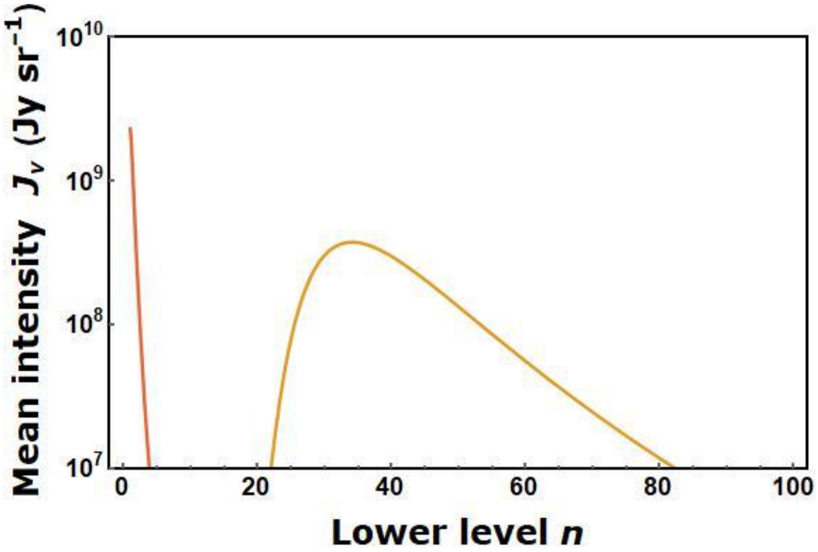
Mean intensity J_ν (Jy sr^{-1})
10^{10}
10^{9}
10^{8}
10^{7}
0
20
40
60
80
100
Lower level *n*

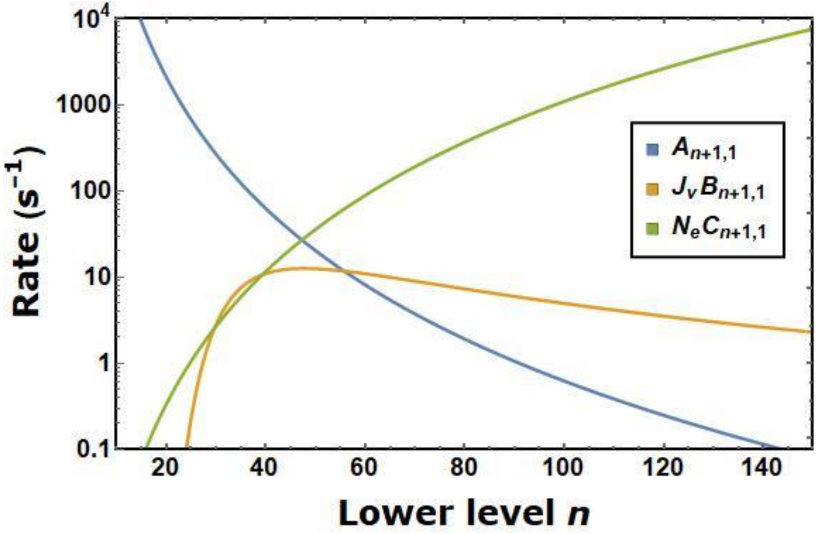
CMB

$$T_{\rm e} = 10^4\,{\rm K}$$

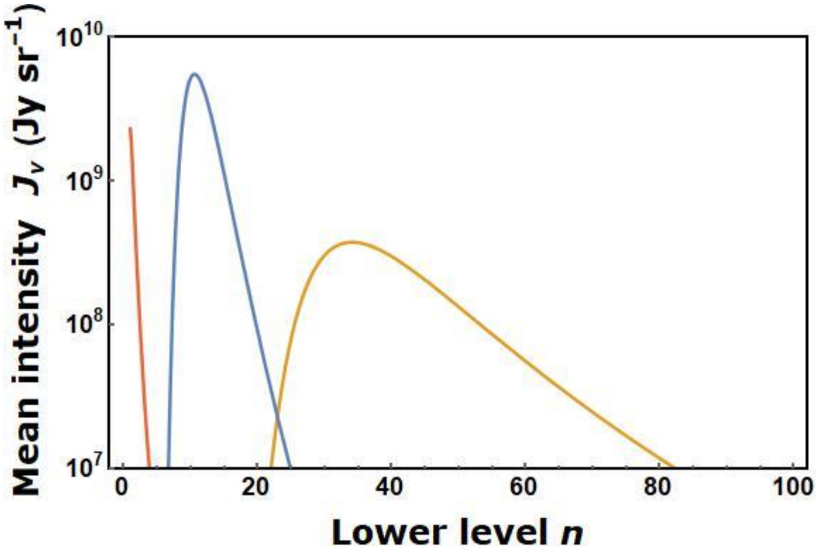
$$N_{\rm e} = 10^2\,{\rm cm}^{-3}$$

$$J_\nu = B_\nu\,(2.7\,{\rm K})$$



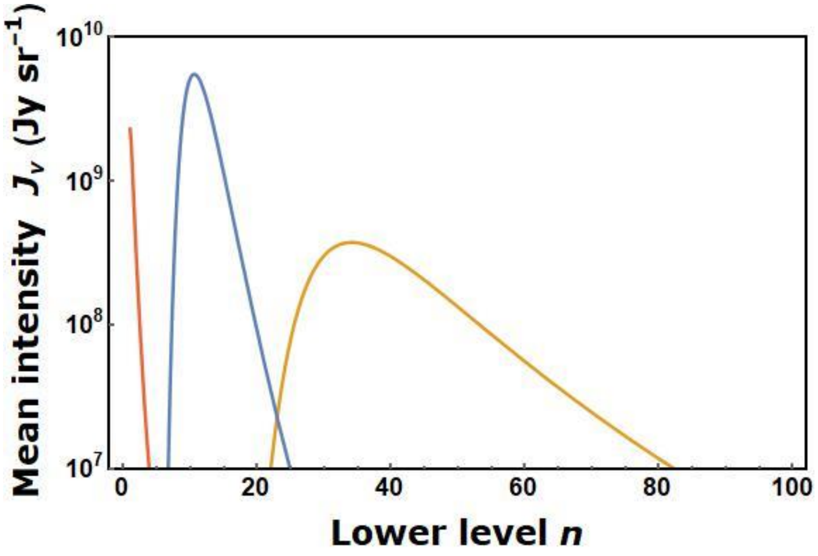


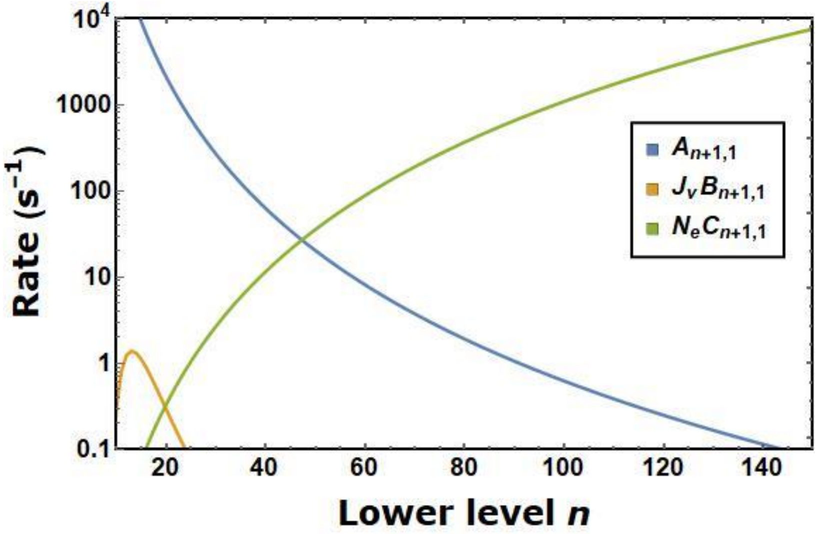
Dust emission



Dust emission

$$T_e = 10^4\,\mathrm{K}$$
$$N_e = 10^2\,\mathrm{cm}^{-3}$$
$$T_d = 50\,\mathrm{K}$$

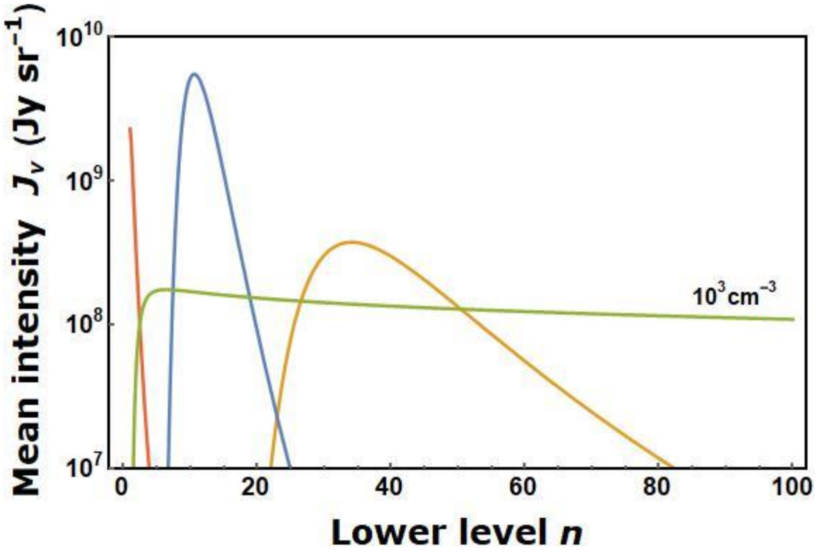




Free-free radiation

$$T_e = 10^4\,\mathrm{K}$$

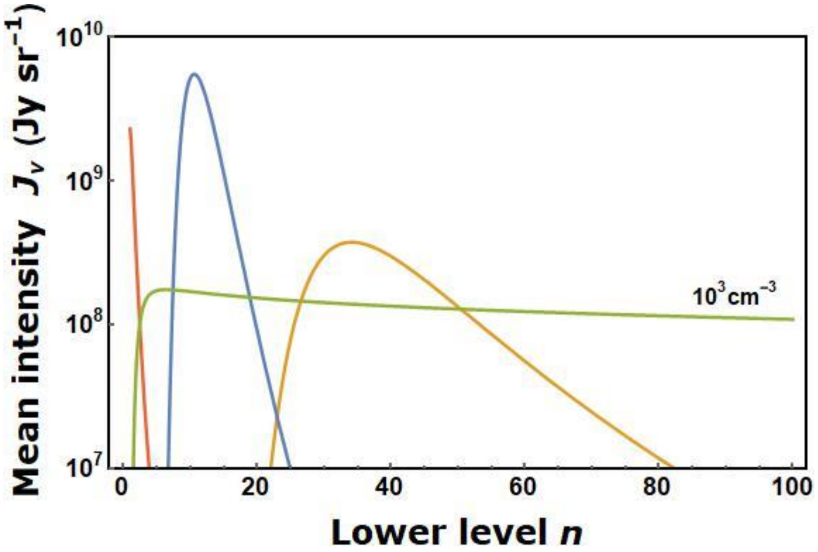
$$N_e = 10^3\,\mathrm{cm}^{-3}$$

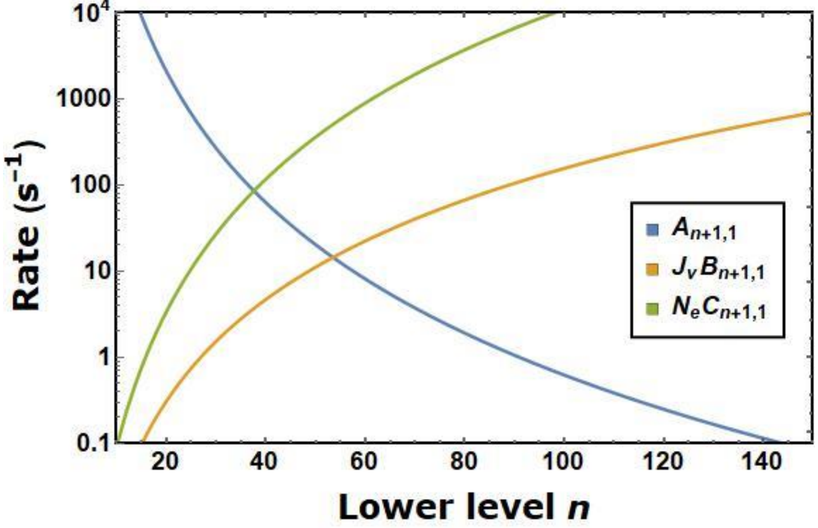


Free-free radiation

$T_e = 10^4\,\mathrm{K}$

$N_e = 10^3\,\mathrm{cm}^{-3}$

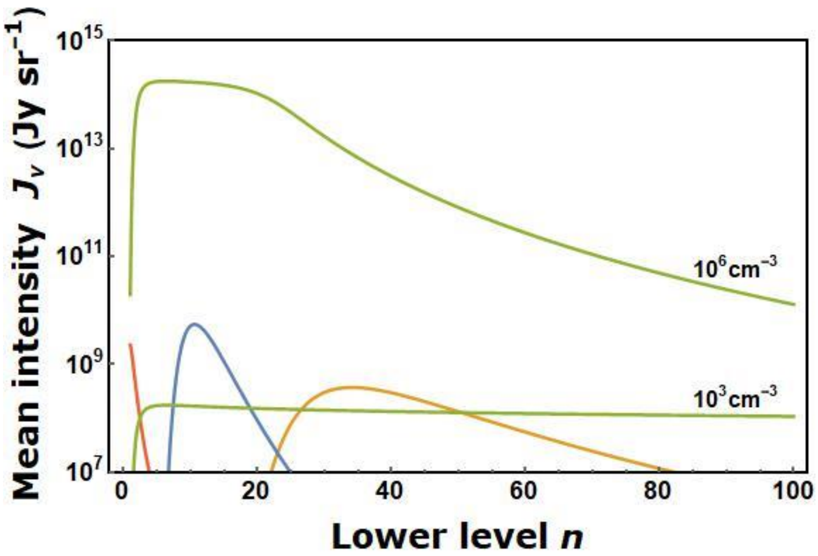


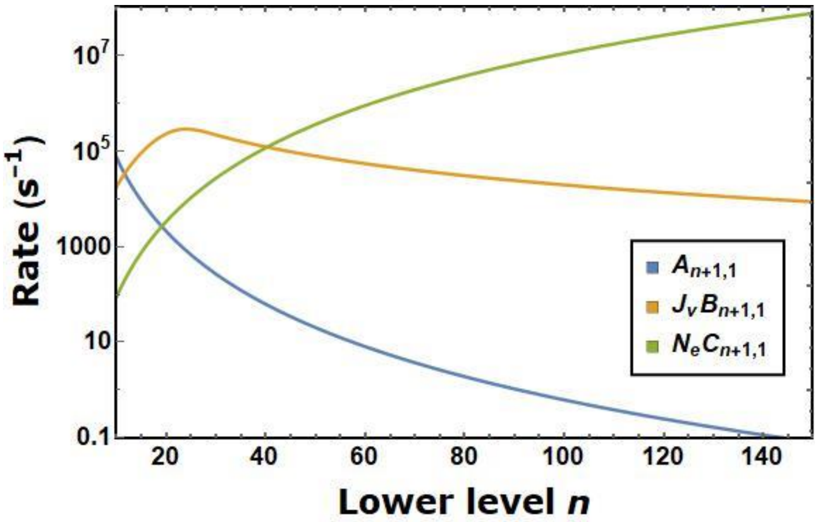


Free-free radiation

$T_e = 10^4\,\mathrm{K}$

$N_e = 10^6\,\mathrm{cm}^{-3}$





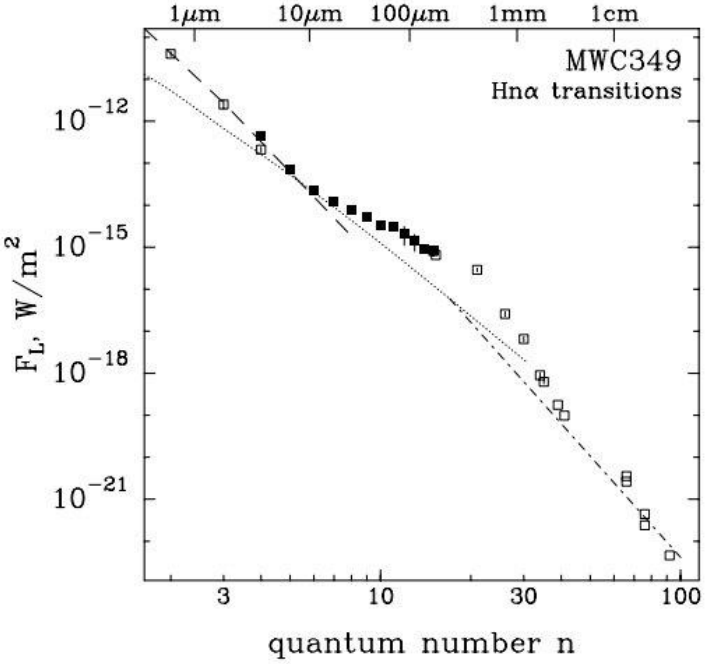
Conclusions

- Be mindful of stopping criteria when using iterative methods
- Inclusion of free-free radiation is important

Prozesky & Smits, 2018, MNRAS, 478, 2766

Future work

- He model
- Recombination line masers of H



Thank you

Any questions?

Prozesky & Smits, 2018, MNRAS, 478, 2766